Angle-Retaining Chromaticity: color invariants and properties

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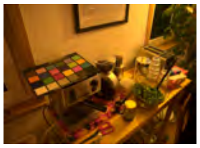
Computational Color Imaging Workshop

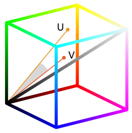
Color constancy: Error evaluation based on angular error

Recovery error:



U
Ground truth

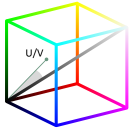
V
Estimated



$$err_{rec} = \arccos\left(\frac{U \cdot V}{\|U\|\|V\|}\right) = \arccos\left(\frac{\sum_i u_i v_i}{\sqrt{\sum_i u_i^2}\sqrt{\sum_i v_i^2}}\right)$$

Reproduction error:

U/V
Reproduction



$$err_{rep} = \arccos\left(\frac{\frac{U}{V} \cdot (1,1,1)}{\|\frac{U}{V}\|\sqrt{3}}\right) = \arccos\left(\frac{\sum_i \frac{u_i}{v_i}}{\sqrt{\sum_i \frac{u_i^2}{v_i^2}}\sqrt{3}}\right)$$

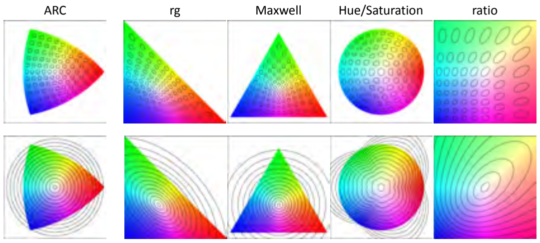
Angle-Retaining Chromaticity diagram

Local distances ≈ Recovery error

Distance from the neutral axis = Reproduction error

[1] M. Buzzelli, S. Bianco and R. Schettini. "ARC: Angle-Retaining Chromaticity diagram for color constancy error analysis." *JOSA A* 37.11 (2020): 1721-1730.

Angle-Retaining Chromaticity diagram

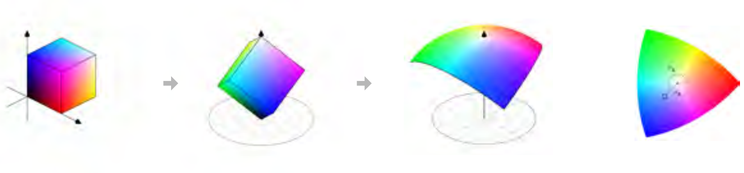


[1] M. Buzzelli, S. Bianco and R. Schettini. "ARC: Angle-Retaining Chromaticity diagram for color constancy error analysis." *JOSA A* 37.11 (2020): 1721-1730.

Angle-Retaining Chromaticity diagram - derivation

$$a_A = \arctan 2\left(\sqrt{3}(G-B), 2R-G-B\right)$$

$$a_R = \arccos\left(\frac{R+G+B}{\sqrt{3}\sqrt{R^2+G^2+B^2}}\right)$$



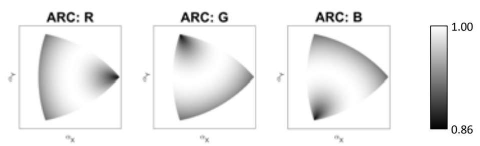
RGB → Rotation and shift to polar coordinates → Radial expansion and flattening / Dropping radial distance

[1] M. Buzzelli, S. Bianco and R. Schettini. "ARC: Angle-Retaining Chromaticity diagram for color constancy error analysis." *JOSA A* 37.11 (2020): 1721-1730.
[2] T. Chen, Z.Deng and J. Ma. "A spherical perceptual color model." *Color Imaging XVIII: Displaying, Processing, Hardcopy, and Applications*. Vol. 8652. SPIE, 2013.

Angular distance vs. Euclidean distance

- If I change the input RGB by a certain amount,
  - how much does the angular distance in RGB change?
  - how much does the euclidean distance in ARC change?

$$\frac{\text{ang\_dist}([R\ G\ B]\oplus\delta, [R\ G\ B]\ominus\delta)}{\text{euc\_dist}(\text{ARC}([R\ G\ B]\oplus\delta), \text{ARC}([R\ G\ B]\ominus\delta))}$$



Angular distance vs. Euclidean distance

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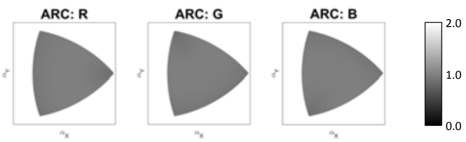
$$\frac{\text{ang\_dist}([R\ G\ B]\oplus\delta, [R\ G\ B]\ominus\delta)}{\text{euc\_dist}(\text{ARC}([R\ G\ B]\oplus\delta), \text{ARC}([R\ G\ B]\ominus\delta))}$$

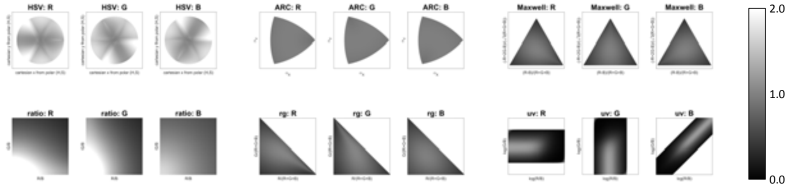


Normalized with a common scale s.t. the center of the diagram is 1:1

Angular distance vs. Euclidean distance

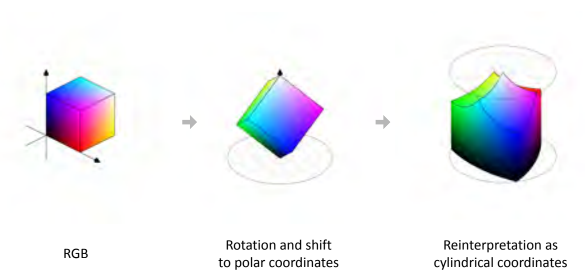
- If I change the input RGB by a certain amount,
  - how much does the angular distance in RGB change?
  - how much does the euclidean distance in *** change?

$$\frac{\text{ang\_dist}([R\ G\ B]\oplus\delta, [R\ G\ B]\ominus\delta)}{\text{euc\_dist}(***([R\ G\ B]\oplus\delta), \text{ARC}([R\ G\ B]\ominus\delta))}$$



Normalized with a common scale s.t. the center of each diagram is 1:1

Angle-Retaining Color space - derivation



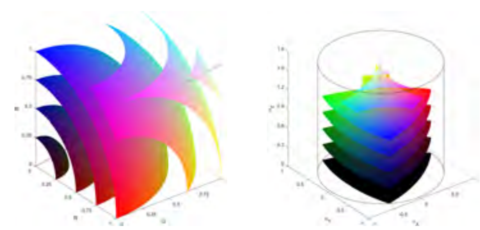
$$\alpha_A = \arctan 2\left(\sqrt{3}(G - B), 2R - G - B\right)$$

$$\alpha_R = \arccos\left(\frac{R + G + B}{\sqrt{3}\sqrt{R^2 + G^2 + B^2}}\right)$$

$$\alpha_Z = \sqrt{R^2 + G^2 + B^2}$$

[3] M. Buzzelli, S. Bianco, R. Schettini. "Angle-Retaining Color Space for Color Data Visualization and Analysis". In *Proceedings of the International Colour Association (AIC) Conference 2021*. Milan, Italy. AIC, pp. 245-250, 2021.

RGB gamut in ARC space



- Concentric spheres in RGB -> Equal-sized horizontal planes in ARC
  - Low-intensity RGB triplets stretched out to span the same area of high-intensity ones
- Rays from the origin in RGB -> Parallel vertical lines in ARC

[3] M. Buzzelli, S. Bianco, R. Schettini. "Angle-Retaining Color Space for Color Data Visualization and Analysis". In *Proceedings of the International Colour Association (AIC) Conference 2021*. Milan, Italy. AIC, pp. 245-250, 2021.

Color invariants of ARC space components

- Which ARC components are invariant to which color transformations [4] ?

	Light intensity change $\begin{pmatrix}a&0&0\\0&a&0\\0&0&a\end{pmatrix}\begin{pmatrix}R\\G\\B\end{pmatrix}$	Light intensity shift $\begin{pmatrix}R\\G\\B\end{pmatrix}+\begin{pmatrix}o_1\\o_1\\o_1\end{pmatrix}$	Light intensity change and shift $\begin{pmatrix}a&0&0\\0&a&0\\0&0&a\end{pmatrix}\begin{pmatrix}R\\G\\B\end{pmatrix}+\begin{pmatrix}o_1\\o_1\\o_1\end{pmatrix}$	Light color change $\begin{pmatrix}a&0&0\\0&b&0\\0&0&c\end{pmatrix}\begin{pmatrix}R\\G\\B\end{pmatrix}$	Light color change and shift $\begin{pmatrix}a&0&0\\0&b&0\\0&0&c\end{pmatrix}\begin{pmatrix}R\\G\\B\end{pmatrix}+\begin{pmatrix}o_1\\o_2\\o_3\end{pmatrix}$
α_A (Hue)	Invariant	Invariant	Invariant	Not invariant	Not invariant
α_R (Saturation)	Invariant	Not invariant	Not invariant	Not invariant	Not invariant
α_Z (Intensity)	Not invariant	Not invariant	Not invariant	Not invariant	Not invariant
	Shadows and shading (no-colored lighting geometry changes)	Diffuse lighting, object highlights, interreflections, infrared sensitivity		Change in illuminant color, light scattering (von Kries)	Increased diffuse light

[4] K. van de Sande, T. Gevers and C. Snoek, "Evaluating Color Descriptors for Object and Scene Recognition," in *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 32, no. 9, pp. 1582-1596, Sept. 2010, doi: 10.1109/TPAMI.2009.154.

Potential for application

- Color constancy
  - A color space that encodes angular distances as Euclidean distances
    - Implicitly provides a sensitivity to angular target metrics.
    - Facilitates optimization process for illuminant estimation methods.
- Texture analysis
  - Inclusion of color-aware descriptors in traditional LBP texture descriptors
    - Improves image classification under varying illuminant conditions
  - Other descriptors potentially enhanced with color-invariant properties
- Image denoising and enhancement
  - Separation of image data into color-related and intensity-related components
    - Successfully applied for image denoising and enhancement
  - Potential benefit in applications where the illuminant characterization should be preserved